Find the area between the curves $y = 3x^2 - 6x$ and $y = 2x + 3$ over the interval $x \in [1, 4]$. **SCORE: _____ / 25 PTS**

**Your final answer must be a number, not an integral. HINT: The answer is NOT 6.**

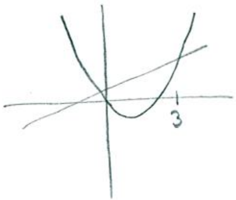
$3x^2 - 6x = 2x + 3$

$3x^2 - 8x - 3 = 0$ 3

$(3x+1)(x-3) = 0$

$x = -\frac{1}{3}, 3$ 3

$\in [1,4]$




2

2 $\int_1^3 (2x+3-(3x^2-6x))\,dx + \int_3^4 (3x^2-6x-(2x+3))\,dx$

3 3

$= \int_1^3 (-3x^2+8x+3)\,dx + \int_3^4 (3x^2-8x-3)\,dx$

$= (-x^3+4x^2+3x)\Big|_1^3 + (x^3-4x^2-3x)\Big|_3^4$ 6

$= -(27-1)+4(9-1)+3(3-1)$

$+(64-27)-4(16-9)-3(4-3)$

$= 11 + 4 + 3$

$= 18$ 3

Find the area of the surface created by revolving the arc of $f(x)=\dfrac{2x^6+1}{8x^2}$ on $[1, 2]$ about the $y-$axis. **SCORE: _____ / 25 PTS**

**Your final answer must be a number, not an integral.**

$= \frac{1}{4}x^4 + \frac{1}{8}x^{-2}$ $\quad f'(x) = x^3 - \frac{1}{4}x^{-3}$ ④

$$\int_1^2 2\pi x\sqrt{1+(x^3-\tfrac{1}{4}x^{-3})^2}\,dx$$ ② ③ ③

$$= 2\pi\int_1^2 x\sqrt{1+x^6-\tfrac{1}{2}+\tfrac{1}{16}x^{-6}}\,dx$$

$$= 2\pi\int_1^2 x\sqrt{x^6+\tfrac{1}{2}+\tfrac{1}{16}x^{-6}}\,dx$$

$$= 2\pi\int_1^2 x(x^3+\tfrac{1}{4}x^{-3})\,dx$$ ④

$$= 2\pi\int_1^2 (x^4+\tfrac{1}{4}x^{-2})\,dx$$

$$= 2\pi\left(\tfrac{1}{5}x^5-\tfrac{1}{4}x^{-1}\right)\Big|_1^2$$ ⑥

$$= 2\pi\left(\tfrac{1}{5}(32-1)-\tfrac{1}{4}(\tfrac{1}{2}-1)\right)$$

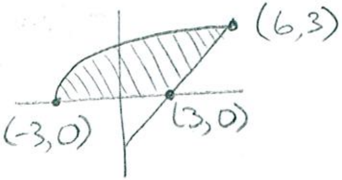
$$= 2\pi\left(\tfrac{31}{5}+\tfrac{1}{8}\right)$$

$$= 2\pi\cdot\tfrac{253}{40}$$

$$= \frac{253\pi}{20}$$ ③

Consider the region defined by $y \le \sqrt{x+3}$, $y \ge x-3$ and $y \ge 0$. **SCORE: _____ / 35 PTS**

[a] Write, **BUT DO NOT EVALUATE**, a **single** integral for the volume of the resulting solid if the region is revolved around the line $x = 10$.




SLICE HORIZONTALLY FOR 1 INTEGRAL

AXIS OF REVOLUTION VERTICAL

WASHER METHOD

$\sqrt{x+3} = x-3$

$x+3 = x^2-6x+9$

$0 = x^2-7x+6$ 3

$= (x-1)(x-6)$

$x = 1$ or $6$

$\downarrow$

$y = \cancel{-2}$ or $3$ 2

$y = \sqrt{x+3} \to x = y^2-3$

$y = x-3 \to x = y+3$ 4

$$\pi \int_0^3 \left[ (10-(y^2-3))^2 - (10-(y+3))^2 \right] dy$$

2 4 3 3

[b] Suppose the region is the base of a solid. Cross sections of the solid perpendicular to the $x-$ axis are semicircles. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$$\frac{\pi}{8} \left[ \int_{-3}^{3} (\sqrt{x+3} - 0)^2 \, dx + \int_3^6 (\sqrt{x+3} - (x-3))^2 \, dx \right]$$

3 3 2 3 3

A solid of revolution has volume $2\pi\int_2^6 (y+6)(\sqrt{2y}-\frac{y}{2})^2\,dy$.

**SCORE: _____ / 15 PTS**

Find the equation of the axis of revolution, and the equations of the boundaries of the region being revolved.

**Sketch & shade in the region being revolved.**

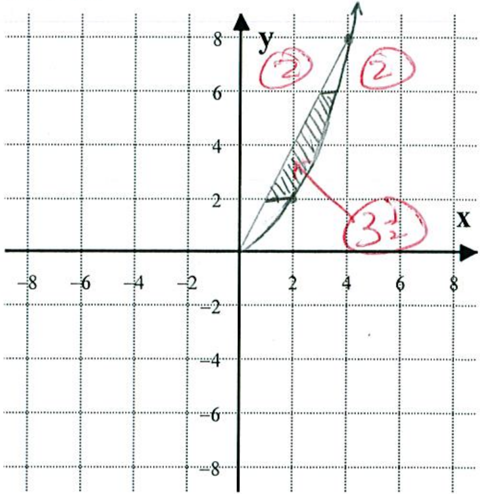
Equation of axis of revolution: $1\frac{1}{2}$ $y=-6$

Equations of boundaries: $1\frac{1}{2}$ $y=2$

$1\frac{1}{2}$ $y=6$

$1\frac{1}{2}$ $x=\sqrt{2y}$ OR $y=\frac{1}{2}x^2$

$1\frac{1}{2}$ $x=\frac{y}{2}$ OR $y=2x$



Find the value of $c$ guaranteed by the Mean Value Theorem for Integrals for $f(x)=3x^2-4x$ on $[-2,1]$. **SCORE: _____ / 20 PTS**

$$f_{AVE} = \frac{\int_{-2}^{1}(3x^2-4x)\,dx}{1-(-2)} \quad 5$$

$$= \frac{(x^3-2x^2)\Big|_{-2}^{1}}{3} \quad 4$$

$$= \frac{(1-{}^-8)-2(1-4)}{3}$$

$$= 5 \quad 3$$

$$f(c) = 3c^2-4c = 5 \quad 3$$

$$3c^2-4c-5=0$$

$$c = \frac{4 \pm \sqrt{16+60}}{6}$$

$$= \frac{4 \pm \sqrt{76}}{6}$$

$$= \frac{2 \pm \sqrt{19}}{3} \quad 3$$

$$\frac{2+\sqrt{19}}{3} > \frac{2+4}{3} = 2$$

$$c = \frac{2-\sqrt{19}}{3} \quad 2$$

An alarm clock has a randomized snooze feature. When it rings, if you hit the snooze button to silence it, the alarm waits a random number of minutes ( $X$ ), then rings again. The probability density function for $X$ is **SCORE: _____ / 30 PTS**

$$f(x) = \begin{cases} k\sqrt[3]{x}, & x \in [1, 8] \\ 0, & x \notin [1, 8] \end{cases}$$ (for some appropriate constant $k$ ).

[a] Find the probability that the alarm rings again less than $4$ minutes after you hit the snooze button.
**Your final answer must be a number, not an integral.**

$\underbrace{\int_1^8 kx^{\frac{1}{3}}\,dx = 1}$

$k \cdot \underbrace{\frac{3}{4}x^{\frac{4}{3}}}\Big|_1^8 = 1$

$k \cdot \frac{3}{4}(16-1) = 1$

$k = \underbrace{\frac{4}{45}}$ 3

$\underbrace{\int_1^4 \frac{4}{45}x^{\frac{1}{3}}\,dx} = \frac{\cancel{4}}{\cancel{45}_{15}} \cdot \frac{3}{\cancel{4}}x^{\frac{4}{3}}\Big|_1^4$

$= \underbrace{\frac{1}{15}(4\sqrt[3]{4}-1)}$ 3

4 POINTS EACH
EXCEPT AS INDICATED

[b] Find the median number of minutes before the alarm rings again.

$\underbrace{\int_1^M \frac{4}{45}x^{\frac{1}{3}}\,dx = \frac{1}{2}}$

$\frac{\cancel{4}}{\cancel{45}_{15}} \cdot \frac{\cancel{3}}{\cancel{4}}x^{\frac{4}{3}}\Big|_1^M = \frac{1}{2}$

$\underbrace{M^{\frac{4}{3}} - 1 = \frac{15}{2}}$

$\underbrace{M = \left(\frac{17}{2}\right)^{\frac{3}{4}}}$ MINUTES